

Code Generation for Data Processing

Lecture 8: Register Allocation

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Winter 2023/24

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- ▶ Map unlimited/virtual registers to limited/architectural registers

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 - ▶ Stack *spilling* – save value register from to stack memory

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 - ▶ Stack *spilling* – save value register from to stack memory
- ▶ ϕ -nodes: ensure all inputs are assigned to same location

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- ▶ Map unlimited/virtual registers to limited/architectural registers
- ▶ Assign a register to every value
 - ▶ Outputs get a (new) register, input operands often require registers
- ▶ When running out of registers, move values to stack
 - ▶ Stack *spilling* – save value register from to stack memory
- ▶ ϕ -nodes: ensure all inputs are assigned to same location
- ▶ Goal: produce correct code, minimize extra load/stores
 - ▶ Regalloc affects performance in orders of magnitude

Register Allocation: Overview Example

```
gauss(%0) {  
  %2 = SUBXri %0, 1  
  %3 = MADDXrrr %0, %2, 0  
  %4 = MOVXconst 2  
  %5 = SDIVrr %3, %4  
  ret %5  
}
```

```
gauss(%0 : X0) {  
  %2 = SUBXri %0, 1 : X1  
  %3 = MADDXrrr %0, %2, 0 : X0  
  %4 = MOVXconst 2 : X1  
  %5 = SDIVrr %3, %4 : X0  
  ret %5  
}
```

- ▶ May also insert copy and stack spilling instructions

Simplest thing that could possibly work

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- ▶ Idea: allocate a one stack slot for every SSA variable/argument

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- ▶ Idea: allocate a one stack slot for every SSA variable/argument
- ▶ Load all instruction operands into registers right before
- ▶ Perform instruction
- ▶ Write result back to stack slot for that SSA variable

Simplest thing that could possibly work

- ▶ Idea: allocate a one stack slot for every SSA variable/argument
 - ▶ Load all instruction operands into registers right before
 - ▶ Perform instruction
 - ▶ Write result back to stack slot for that SSA variable
- + Simple, always works, debugging easy
- *Extremely* inefficient in time and space

Regalloc Example 1

```
gauss(%0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

Regalloc Example 1

```
gauss(%0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0 : X0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

Regalloc Example 1

```
gauss(%0)
  %2 = SUBXri %0, 1
  %3 = MADDXrrr %0, %2, 0
  %4 = MOVXconst 2
  %5 = SDIVrr %3, %4
  ret %5
```

```
gauss(%0 : X0)
  %spills = alloca 8
  STRXi %0, %spills, 0

  %2 = SUBXri %0, 1

  %3 = MADDXrrr %0, %2, 0

  %4 = MOVXconst 2

  %5 = SDIVrr %3, %4

  ret %5
```

Regalloc Example 1

```
gauss(%0)
  %2 = SUBXri %0, 1
  %3 = MADDXrrr %0, %2, 0
  %4 = MOVXconst 2
  %5 = SDIVrr %3, %4
  ret %5
```

```
gauss(%0 : X0)
  %spills = alloca 8
  STRXi %0, %spills, 0
  %10 = LDRXi %spills, 0 : X0
  %2 = SUBXri %10, 1

  %3 = MADDXrrr %0, %2, 0

  %4 = MOVXconst 2

  %5 = SDIVrr %3, %4

  ret %5
```

Regalloc Example 1

```
gauss(%0)
  %2 = SUBXri %0, 1
  %3 = MADDXrrr %0, %2, 0
  %4 = MOVXconst 2
  %5 = SDIVrr %3, %4
  ret %5
```

```
gauss(%0 : X0)
  %spills = alloca 8
  STRXi %0, %spills, 0
  %10 = LDRXi %spills, 0 : X0
  %2 = SUBXri %10, 1 : X0

  %3 = MADDXrrr %0, %2, 0

  %4 = MOVXconst 2

  %5 = SDIVrr %3, %4

  ret %5
```


Regalloc Example 1

```
gauss(%0)
  %2 = SUBXri %0, 1
  %3 = MADDXrrr %0, %2, 0
  %4 = MOVXconst 2
  %5 = SDIVrr %3, %4
  ret %5
```

```
gauss(%0 : X0)
  %spills = alloca 16
  STRXi %0, %spills, 0
  %10 = LDRXi %spills, 0 : X0
  %2 = SUBXri %10, 1 : X0
  STRXi %2, %spills, 8

  %3 = MADDXrrr %0, %2, 0

  %4 = MOVXconst 2

  %5 = SDIVrr %3, %4

  ret %5
```

Regalloc Example 1

```
gauss(%0)
  %2 = SUBXri %0, 1
  %3 = MADDXrrr %0, %2, 0
  %4 = MOVXconst 2
  %5 = SDIVrr %3, %4
  ret %5
```

```
gauss(%0 : X0)
  %spills = alloca 16
  STRXi %0, %spills, 0
  %10 = LDRXi %spills, 0 : X0
  %2 = SUBXri %10, 1 : X0
  STRXi %2, %spills, 8
  %11 = LDRXi %spills, 0 : X0
  %12 = LDRXi %spills, 8 : X1
  %3 = MADDXrrr %0, %2, 0

  %4 = MOVXconst 2

  %5 = SDIVrr %3, %4

  ret %5
```

Regalloc Example 1

```
gauss(%0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0 : X0)
```

```
  %spills = alloca 16
```

```
  STRXi %0, %spills, 0
```

```
  %10 = LDRXi %spills, 0 : X0
```

```
  %2 = SUBXri %10, 1 : X0
```

```
  STRXi %2, %spills, 8
```

```
  %11 = LDRXi %spills, 0 : X0
```

```
  %12 = LDRXi %spills, 8 : X1
```

```
  %3 = MADDXrrr %11, %12, 0 : X0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

Regalloc Example 1

```
gauss(%0)
```

```
    %2 = SUBXri %0, 1
```

```
    %3 = MADDXrrr %0, %2, 0
```

```
    %4 = MOVXconst 2
```

```
    %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0 : X0)
```

```
    %spills = alloca 24
```

```
    STRXi %0, %spills, 0
```

```
    %10 = LDRXi %spills, 0 : X0
```

```
    %2 = SUBXri %10, 1 : X0
```

```
    STRXi %2, %spills, 8
```

```
    %11 = LDRXi %spills, 0 : X0
```

```
    %12 = LDRXi %spills, 8 : X1
```

```
    %3 = MADDXrrr %11, %12, 0 : X0
```

```
    STRXi %3, %spills, 16
```

```
    %4 = MOVXconst 2
```

```
    %5 = SDIVrr %3, %4
```

```
ret %5
```

Regalloc Example 1

```
gauss(%0)
```

```
    %2 = SUBXri %0, 1
```

```
    %3 = MADDXrrr %0, %2, 0
```

```
    %4 = MOVXconst 2
```

```
    %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0 : X0)
```

```
    %spills = alloca 24
```

```
    STRXi %0, %spills, 0
```

```
    %10 = LDRXi %spills, 0 : X0
```

```
    %2 = SUBXri %10, 1 : X0
```

```
    STRXi %2, %spills, 8
```

```
    %11 = LDRXi %spills, 0 : X0
```

```
    %12 = LDRXi %spills, 8 : X1
```

```
    %3 = MADDXrrr %11, %12, 0 : X0
```

```
    STRXi %3, %spills, 16
```

```
    %4 = MOVXconst 2 : X0
```

```
    %5 = SDIVrr %3, %4
```

```
ret %5
```

Regalloc Example 1

```
gauss(%0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0 : X0)
```

```
  %spills = alloca 32
```

```
  STRXi %0, %spills, 0
```

```
  %10 = LDRXi %spills, 0 : X0
```

```
  %2 = SUBXri %10, 1 : X0
```

```
  STRXi %2, %spills, 8
```

```
  %11 = LDRXi %spills, 0 : X0
```

```
  %12 = LDRXi %spills, 8 : X1
```

```
  %3 = MADDXrrr %11, %12, 0 : X0
```

```
  STRXi %3, %spills, 16
```

```
  %4 = MOVXconst 2 : X0
```

```
  STRXi %4,i %spills, 24
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

Regalloc Example 1

```
gauss(%0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0 : X0)
```

```
  %spills = alloca 32
```

```
  STRXi %0, %spills, 0
```

```
  %10 = LDRXi %spills, 0 : X0
```

```
  %2 = SUBXri %10, 1 : X0
```

```
  STRXi %2, %spills, 8
```

```
  %11 = LDRXi %spills, 0 : X0
```

```
  %12 = LDRXi %spills, 8 : X1
```

```
  %3 = MADDXrrr %11, %12, 0 : X0
```

```
  STRXi %3, %spills, 16
```

```
  %4 = MOVXconst 2 : X0
```

```
  STRXi %4,i %spills, 24
```

```
  %13 = LDRXi %spills, 16 : X0
```

```
  %14 = LDRXi %spills, 24 : X1
```

```
  %5 = SDIVrr %13, %14
```

```
ret %5
```

Regalloc Example 1

```
gauss(%0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0 : X0)
```

```
  %spills = alloca 32
```

```
  STRXi %0, %spills, 0
```

```
  %10 = LDRXi %spills, 0 : X0
```

```
  %2 = SUBXri %10, 1 : X0
```

```
  STRXi %2, %spills, 8
```

```
  %11 = LDRXi %spills, 0 : X0
```

```
  %12 = LDRXi %spills, 8 : X1
```

```
  %3 = MADDXrrr %11, %12, 0 : X0
```

```
  STRXi %3, %spills, 16
```

```
  %4 = MOVXconst 2 : X0
```

```
  STRXi %4,i %spills, 24
```

```
  %13 = LDRXi %spills, 16 : X0
```

```
  %14 = LDRXi %spills, 24 : X1
```

```
  %5 = SDIVrr %13, %14 : X0
```

```
ret %5
```


Regalloc Example 1

```
gauss(%0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0 : X0)
```

```
  %spills = alloca 40
```

```
  STRXi %0, %spills, 0
```

```
  %10 = LDRXi %spills, 0 : X0
```

```
  %2 = SUBXri %10, 1 : X0
```

```
  STRXi %2, %spills, 8
```

```
  %11 = LDRXi %spills, 0 : X0
```

```
  %12 = LDRXi %spills, 8 : X1
```

```
  %3 = MADDXrrr %11, %12, 0 : X0
```

```
  STRXi %3, %spills, 16
```

```
  %4 = MOVXconst 2 : X0
```

```
  STRXi %4,i %spills, 24
```

```
  %13 = LDRXi %spills, 16 : X0
```

```
  %14 = LDRXi %spills, 24 : X1
```

```
  %5 = SDIVrr %13, %14 : X0
```

```
  STRXi %5, %spills, 32
```

```
ret %5
```

Regalloc Example 1

```
gauss(%0)
```

```
  %2 = SUBXri %0, 1
```

```
  %3 = MADDXrrr %0, %2, 0
```

```
  %4 = MOVXconst 2
```

```
  %5 = SDIVrr %3, %4
```

```
ret %5
```

```
gauss(%0 : X0)
```

```
  %spills = alloca 40
```

```
  STRXi %0, %spills, 0
```

```
  %10 = LDRXi %spills, 0 : X0
```

```
  %2 = SUBXri %10, 1 : X0
```

```
  STRXi %2, %spills, 8
```

```
  %11 = LDRXi %spills, 0 : X0
```

```
  %12 = LDRXi %spills, 8 : X1
```

```
  %3 = MADDXrrr %11, %12, 0 : X0
```

```
  STRXi %3, %spills, 16
```

```
  %4 = MOVXconst 2 : X0
```

```
  STRXi %4,i %spills, 24
```

```
  %13 = LDRXi %spills, 16 : X0
```

```
  %14 = LDRXi %spills, 24 : X1
```

```
  %5 = SDIVrr %13, %14 : X0
```

```
  STRXi %5, %spills, 32
```

```
  %15 = LDRXi %spills, 32 : X0
```

```
ret %15
```

Handling PHI Nodes

- ▶ ϕ -node needs to become register or stack slot
 - ▶ Simplest thing that could possibly work: PHI becomes stack slot
- ▶ Remember: ϕ -nodes are executed on the edge

Handling PHI Nodes

- ▶ ϕ -node needs to become register or stack slot
 - ▶ Simplest thing that could possibly work: PHI becomes stack slot
- ▶ Remember: ϕ -nodes are executed on the edge
- ▶ Idea: predecessors write their value to that location at the end
 - ▶ First pass: define/allocate storage for ϕ -node, but ignore inputs
 - ▶ Second pass: insert move operations at end of predecessors

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 1

```
identity(%0)

  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]

  %4 = ADDXri %3, 1

  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 1

```
identity(%0 : X0)
  %spills = alloca 8
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]

  %4 = ADDXri %3, 1

  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 1

```
identity(%0 : X0)
  %spills = alloca 16
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]

  %4 = ADDXri %3, 1

  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 1

```
identity(%0 : X0)
  %spills = alloca 16
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0

  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```


Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 1

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16

  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 1

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %2, %6
6:
  ret %3
```

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 1

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %2, %6
6:%13 = LDRXi %spills, 8 : X0
  ret %13
```

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 2

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %2, %6
6:%13 = LDRXi %spills, 8 : X0
  ret %13
```

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 2

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  %c0 = MOVXconst 0 : X0
  STRXi %c, %spills, 8
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %2, %6
6:%13 = LDRXi %spills, 8 : X0
  ret %13
```

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

Pass 2

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  %c0 = MOVXconst 0 : X0
  STRXi %c, %spills, 8
  br %2
2: %3 = phi [ 0, %1 ], [ %4, %2 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %14 = LDRXi %spills, 16 : X0
  STRXi %14, %spills, 8
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %2, %6
6: %13 = LDRXi %spills, 8 : X0
  ret %13
```

Regalloc Example 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  %c0 = MOVXconst 0 : X0
  STRXi %c, %spills, 8
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %14 = LDRXi %spills, 16 : X0
  STRXi %14, %spills, 8
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %2, %6
6:%13 = LDRXi %spills, 8 : X0
  ret %13
```

Regalloc Example 2

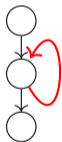
```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %2 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %2, %6
6:
  ret %3
```

► Original value lost in %6!

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  %c0 = MOVXconst 0 : X0
  STRXi %c, %spills, 8
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %2 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %14 = LDRXi %spills, 16 : X0
  STRXi %14, %spills, 8
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %2, %6
6:%13 = LDRXi %spills, 8 : X0
  ret %13
```

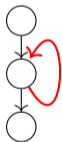

Critical Edges

- ▶ Critical edge: edge from block with mult. succs. to block with mult. preds.
- ▶ Problem: cannot place move on such edges
 - ▶ When placing in predecessor, they would also execute for other successor
⇒ unnecessary and – worse – incorrect



Critical Edges

- ▶ Critical edge: edge from block with mult. succs. to block with mult. preds.
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⇒ unnecessary and – worse – incorrect



- ▶ *Break* critical edges: insert an empty block

Critical Edges

- ▶ Critical edge: edge from block with mult. succs. to block with mult. preds.
- ▶ Problem: cannot place move on such edges
 - ▶ When placing in predecessor, they would also execute for other successor
⇒ unnecessary and – worse – incorrect



- ▶ *Break* critical edges: insert an empty block

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

```
identity(%0)

  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]

  %4 = ADDXri %3, 1

  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:br %2
7:
  ret %3
```

Pass 1

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

```
identity(%0 : X0)
  %spills = alloca 8
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]

  %4 = ADDXri %3, 1

  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:br %2
7:
  ret %3
```

Pass 1

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

```
identity(%0 : X0)
  %spills = alloca 16
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]

  %4 = ADDXri %3, 1

  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:br %2
7:
  ret %3
```

Pass 1

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

```
identity(%0 : X0)
  %spills = alloca 16
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0

  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:br %2
7:
  ret %3
```

Pass 1

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16

  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:br %2
7:
  ret %3
```

Pass 1

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %6, %7
6:br %2
7:
  ret %3
```

Pass 1

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %6, %7
6:br %2
7:%13 = LDRXi %spills, 8 : X0
  ret %13
```

Pass 1

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %6, %7
6:br %2
7:%13 = LDRXi %spills, 8 : X0
  ret %13
```

Pass 2

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

Pass 2

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  %c0 = MOVXconst 0 : X0
  STRXi %c, %spills, 8
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %6, %7
6:br %2
7:%13 = LDRXi %spills, 8 : X0
  ret %13
```

Regalloc Example 2 – Attempt 2

```
identity(%0)
  br %2
2:
  %3 = phi [ 0, %1 ], [ %4, %6 ]
  %4 = ADDXri %3, 1
  %5 = CMPXrr_BLS %4, %0
  br %5, %6, %7
6:
  br %2
7:
  ret %3
```

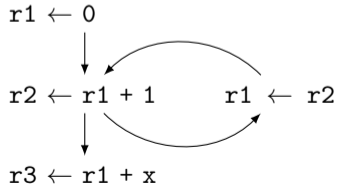
Pass 2

```
identity(%0 : X0)
  %spills = alloca 24
  STRXi %0, %spills, 0
  %c0 = MOVXconst 0 : X0
  STRXi %c, %spills, 8
  br %2
2:%3 = phi [ 0, %1 ], [ %4, %6 ]
  %10 = LDRXi %spills, 8 : X0
  %4 = ADDXri %10, 1 : X0
  STRXi %4, %spills, 16
  %11 = LDRXi %spills, 16 : X0
  %12 = LDRXi %spills, 0 : X1
  %5 = CMPXrr_BLS %11, %12
  br %5, %6, %7
6:%14 = LDRXi %spills, 16 : X0
  STRXi %14, %spills, 8
  br %2
7:%13 = LDRXi %spills, 8 : X0
  ret %13
```

Handling Critical Edges

Breaking Edges

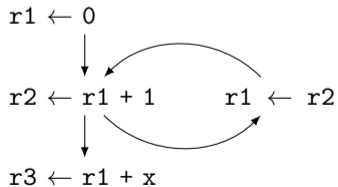
- ▶ Insert new block for moves



Handling Critical Edges

Breaking Edges

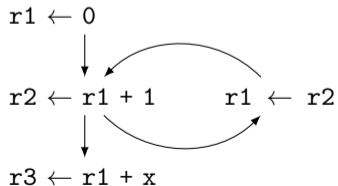
- ▶ Insert new block for moves
- + Simple, no analyses needed
- Bad performance in loops



Handling Critical Edges

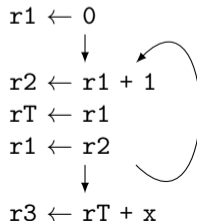
Breaking Edges

- ▶ Insert new block for moves
- + Simple, no analyses needed
- Bad performance in loops



Copy Used Values

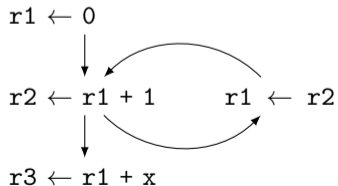
- ▶ Move values still used to new reg.



Handling Critical Edges

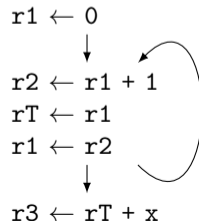
Breaking Edges

- ▶ Insert new block for moves
- + Simple, no analyses needed
- Bad performance in loops



Copy Used Values

- ▶ Move values still used to new reg.
- + Performance might be better
- Needs more registers



Regalloc Example 3

```
odd(%0)
  br %2
2:
  %3 = phi [ %0, %1 ], [ %8, %7 ]
  %4 = phi [ 1, %1 ], [ %5, %7 ]
  %5 = phi [ 0, %1 ], [ %4, %7 ]
  %6 = CBNZX(%3)
  br %6, %7, %9
7:
  %8 = SUBXri %3, 1
  br %2
9:
  ret %4
```

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0

  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32

  br %2
9:%12 = LDRXi %spills, 24 : X0
  ret %12
```

Regalloc Example 3

```
odd(%0)
  br %2
2:
  %3 = phi [ %0, %1 ], [ %8, %7 ]
  %4 = phi [ 1, %1 ], [ %5, %7 ]
  %5 = phi [ 0, %1 ], [ %4, %7 ]
  %6 = CBNZX(%3)
  br %6, %7, %9
7:
  %8 = SUBXri %3, 1
  br %2
9:
  ret %4
```

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0

  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32

  br %2
9:%12 = LDRXi %spills, 24 : X0
  ret %12
```

Regalloc Example 3

```
odd(%0)
  br %2
2:
  %3 = phi [ %0, %1 ], [ %8, %7 ]
  %4 = phi [ 1, %1 ], [ %5, %7 ]
  %5 = phi [ 0, %1 ], [ %4, %7 ]
  %6 = CBNZX(%3)
  br %6, %7, %9
7:
  %8 = SUBXri %3, 1
  br %2
9:
  ret %4
```

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %13 = LDRXi %spills, 0 : X0; STRXi %13, %spills, 8

  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32
  %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8

  br %2
9:%12 = LDRXi %spills, 24 : X0
  ret %12
```

Regalloc Example 3

```
odd(%0)
  br %2
2:
  %3 = phi [ %0, %1 ], [ %8, %7 ]
  %4 = phi [ 1, %1 ], [ %5, %7 ]
  %5 = phi [ 0, %1 ], [ %4, %7 ]
  %6 = CBNZX(%3)
  br %6, %7, %9
7:
  %8 = SUBXri %3, 1
  br %2
9:
  ret %4
```

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %13 = LDRXi %spills, 0 : X0; STRXi %13, %spills, 8

  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32
  %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8

  br %2
9:%12 = LDRXi %spills, 24 : X0
  ret %12
```

Regalloc Example 3

```
odd(%0)
  br %2
2:
  %3 = phi [ %0, %1 ], [ %8, %7 ]
  %4 = phi [ 1, %1 ], [ %5, %7 ]
  %5 = phi [ 0, %1 ], [ %4, %7 ]
  %6 = CBNZX(%3)
  br %6, %7, %9
7:
  %8 = SUBXri %3, 1
  br %2
9:
  ret %4
```

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16

  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %l0 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%l0)
  br %6, %7, %9
7:%l1 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
  %l4 = LDRXi %spills, 40 : X0; STRXi %l4, %spills, 8
  %l5 = LDRXi %spills, 24 : X0; STRXi %l5, %spills, 16

  br %2
9:%l2 = LDRXi %spills, 24 : X0
  ret %l2
```

Regalloc Example 3

```
odd(%0)
  br %2
2:
  %3 = phi [ %0, %1 ], [ %8, %7 ]
  %4 = phi [ 1, %1 ], [ %5, %7 ]
  %5 = phi [ 0, %1 ], [ %4, %7 ]
  %6 = CBNZX(%3)
  br %6, %7, %9
7:
  %8 = SUBXri %3, 1
  br %2
9:
  ret %4
```

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16

  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %l0 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%l0)
  br %6, %7, %9
7:%l1 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
  %l4 = LDRXi %spills, 40 : X0; STRXi %l4, %spills, 8
  %l5 = LDRXi %spills, 24 : X0; STRXi %l5, %spills, 16

  br %2
9:%l2 = LDRXi %spills, 24 : X0
  ret %l2
```

Regalloc Example 3

```
odd(%0)
  br %2
2:
  %3 = phi [ %0, %1 ], [ %8, %7 ]
  %4 = phi [ 1, %1 ], [ %5, %7 ]
  %5 = phi [ 0, %1 ], [ %4, %7 ]
  %6 = CBNZX(%3)
  br %6, %7, %9
7:
  %8 = SUBXri %3, 1
  br %2
9:
  ret %4
```

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
  %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %l0 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%l0)
  br %6, %7, %9
7:%l1 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
  %l4 = LDRXi %spills, 40 : X0; STRXi %l4, %spills, 8
  %l5 = LDRXi %spills, 24 : X0; STRXi %l5, %spills, 16
  %l6 = LDRXi %spills, 16 : X0; STRXi %l6, %spills, 24
  br %2
9:%l2 = LDRXi %spills, 24 : X0
  ret %l2
```


Regalloc Example 3

```
odd(%0)
  br %2
2:
  %3 = phi [ %0, %1 ], [ %8, %7 ]
  %4 = phi [ 1, %1 ], [ %5, %7 ]
  %5 = phi [ 0, %1 ], [ %4, %7 ]
  %6 = CBNZX(%3)
  br %6, %7, %9
7:
  %8 = SUBXri %3, 1
  br %2
9:
  ret %4
```

► Value of ϕ node lost!

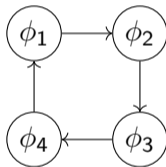
```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
  %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %l0 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%l0)
  br %6, %7, %9
7:%l1 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
  %l4 = LDRXi %spills, 40 : X0; STRXi %l4, %spills, 8
  %l5 = LDRXi %spills, 24 : X0; STRXi %l5, %spills, 16
  %l6 = LDRXi %spills, 16 : X0; STRXi %l6, %spills, 24
  br %2
9:%l2 = LDRXi %spills, 24 : X0
  ret %l2
```

PHI Cycles

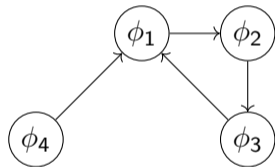
- ▶ Problem: ϕ -nodes can depend on each other
- ▶ Can be chains (ordering matters) or cycles (need to be broken)
- ▶ Note: only ϕ -nodes defined in same block are relevant/problematic



$$\begin{aligned}\phi_1 &= \phi(\phi_2, \dots) \\ \phi_2 &= \phi(\phi_3, \dots) \\ \phi_3 &= \phi(v, \dots)\end{aligned}$$



$$\begin{aligned}\phi_1 &= \phi(\phi_2, \dots) \\ \phi_2 &= \phi(\phi_3, \dots) \\ \phi_3 &= \phi(\phi_4, \dots) \\ \phi_4 &= \phi(\phi_1, \dots)\end{aligned}$$



$$\begin{aligned}\phi_1 &= \phi(\phi_2, \dots) \\ \phi_2 &= \phi(\phi_3, \dots) \\ \phi_3 &= \phi(\phi_1, \dots) \\ \phi_4 &= \phi(\phi_1, \dots)\end{aligned}$$

Handling PHI Cycles

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1. Compute number of other ϕ nodes reading other ϕ on same edge

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2. For each ϕ with 0 readers: handle node/chain
 - ▶ No readers \rightsquigarrow start of chain
 - ▶ Handling node may unblock next element in chain

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3. For all remaining ϕ -nodes: must be cycles, reader count always 1
 - ▶ Choose arbitrary node, load to temporary register, unblock value
 - ▶ Handle just-created chain
 - ▶ Write temporary register to target

Handling PHI Cycles

1. Compute number of other ϕ nodes reading other ϕ on same edge
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3. For all remaining ϕ -nodes: must be cycles, reader count always 1
 - ▶ Choose arbitrary node, load to temporary register, unblock value
 - ▶ Handle just-created chain
 - ▶ Write temporary register to target

Resolving ϕ cycles requires an extra register (or stack slot)

Regalloc Example 3 – Attempt 2

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
```

Edge %1 → %2

Critical ϕ :

```
br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32
```

```
br %2
9:%12 = LDRXi %spills, 24 : X0
ret %12
```


Regalloc Example 3 – Attempt 2

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
```

Edge %1 \rightarrow %2

Critical ϕ :

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2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
%4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
%5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
%10 = LDRXi %spills, 8 : X0
%6 = CBNZX(%10)
br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
%8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32
```

```
br %2
9:%12 = LDRXi %spills, 24 : X0
ret %12
```

Regalloc Example 3 – Attempt 2

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %13 = LDRXi %spills, 0 : X0; STRXi %13, %spills, 8
```

Edge %1 \rightarrow %2

Critical ϕ :

```
br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
%4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
%5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
%10 = LDRXi %spills, 8 : X0
%6 = CBNZX(%10)
br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
%8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32
```

```
br %2
9:%12 = LDRXi %spills, 24 : X0
ret %12
```

Regalloc Example 3 – Attempt 2

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odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %13 = LDRXi %spills, 0 : X0; STRXi %13, %spills, 8
```

Edge %1 → %2

Critical ϕ :

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br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
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  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32
```

```
br %2
9:%12 = LDRXi %spills, 24 : X0
ret %12
```

Regalloc Example 3 – Attempt 2

Edge %1 → %2

Critical ϕ :

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %13 = LDRXi %spills, 0 : X0; STRXi %13, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16

  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
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  %8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32

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```

Regalloc Example 3 – Attempt 2

Edge %1 \rightarrow %2

Critical ϕ :

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  STRXi %0, %spills, 0
  %13 = LDRXi %spills, 0 : X0; STRXi %13, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16

  br %2

2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9

7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32

  br %2

9:%12 = LDRXi %spills, 24 : X0
  ret %12
```

Regalloc Example 3 – Attempt 2

Edge %1 → %2

Critical ϕ :

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
  %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32

br %2
9:%12 = LDRXi %spills, 24 : X0
ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
  %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32

br %2
9:%12 = LDRXi %spills, 24 : X0
ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
  %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
  %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8

  br %2
9:%12 = LDRXi %spills, 24 : X0
  ret %12
```


Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %13 = LDRXi %spills, 0 : X0; STRXi %13, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
  %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %12, 1 : X0; STRXi %8, %spills, 32
  %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8

  br %2
9:%12 = LDRXi %spills, 24 : X0
  ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

▶ %4

```
odd(%0 : X0)
    %spills = alloca 40
    STRXi %0, %spills, 0
    %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
    %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
    %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
    br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
    %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
    %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
    %l0 = LDRXi %spills, 8 : X0
    %6 = CBNZX(%l0)
    br %6, %7, %9
7:%l1 = LDRXi %spills, 8 : X0
    %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
    %l4 = LDRXi %spills, 40 : X0; STRXi %l4, %spills, 8

    br %2
9:%l2 = LDRXi %spills, 24 : X0
    ret %l2
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

▶ %4

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
  %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
  %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8

br %2
9:%12 = LDRXi %spills, 24 : X0
ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

▶ %4

▶ %5

```
odd(%0 : X0)
    %spills = alloca 40
    STRXi %0, %spills, 0
    %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
    %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
    %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
    br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
    %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
    %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
    %10 = LDRXi %spills, 8 : X0
    %6 = CBNZX(%10)
    br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
    %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
    %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8

    br %2
9:%12 = LDRXi %spills, 24 : X0
    ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

▶ %4

▶ %5

```
odd(%0 : X0)
    %spills = alloca 40
    STRXi %0, %spills, 0
    %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
    %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
    %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
    br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
    %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
    %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
    %10 = LDRXi %spills, 8 : X0
    %6 = CBNZX(%10)
    br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
    %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
    %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8

    br %2
9:%12 = LDRXi %spills, 24 : X0
    ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

- ▶ %4 #readers: 1
- ▶ %5 #readers: 1

```
odd(%0 : X0)
    %spills = alloca 40
    STRXi %0, %spills, 0
    %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
    %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
    %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
    br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
    %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
    %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
    %10 = LDRXi %spills, 8 : X0
    %6 = CBNZX(%10)
    br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
    %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
    %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8

    br %2
9:%12 = LDRXi %spills, 24 : X0
    ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

- ▶ %4 #readers: 1 – broken
- ▶ %5 #readers: 1

Action: break %4

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
  %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
  %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8

br %2
9:%12 = LDRXi %spills, 24 : X0
ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

- ▶ %4 #readers: 1 – broken
- ▶ %5 #readers: 0

```
odd(%0 : X0)
  %spills = alloca 40
  STRXi %0, %spills, 0
  %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
  %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
  %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
  br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
  %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
  %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
  %10 = LDRXi %spills, 8 : X0
  %6 = CBNZX(%10)
  br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
  %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
  %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8
  %15 = LDRXi %spills, 24 : X1

  br %2
9:%12 = LDRXi %spills, 24 : X0
  ret %12
```


Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

- ▶ %4 #readers: 1 – broken
- ▶ %5 #readers: 0

```
odd(%0 : X0)
    %spills = alloca 40
    STRXi %0, %spills, 0
    %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
    %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
    %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
    br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
    %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
    %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
    %10 = LDRXi %spills, 8 : X0
    %6 = CBNZX(%10)
    br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
    %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
    %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8
    %15 = LDRXi %spills, 24 : X1

    br %2
9:%12 = LDRXi %spills, 24 : X0
    ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

- ▶ %4 #readers: 0 – broken
- ▶ %5 #readers: 0

```
odd(%0 : X0)
    %spills = alloca 40
    STRXi %0, %spills, 0
    %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
    %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
    %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
    br %2
2:%3 = phi [ %0, %1 ], [ %8, %7 ] // spills+8
    %4 = phi [ 1, %1 ], [ %5, %7 ] // spills+16
    %5 = phi [ 0, %1 ], [ %4, %7 ] // spills+24
    %10 = LDRXi %spills, 8 : X0
    %6 = CBNZX(%10)
    br %6, %7, %9
7:%11 = LDRXi %spills, 8 : X0
    %8 = SUBXri %l2, 1 : X0; STRXi %8, %spills, 32
    %14 = LDRXi %spills, 40 : X0; STRXi %14, %spills, 8
    %15 = LDRXi %spills, 24 : X1
    %16 = LDRXi %spills, 16 : X0; STRXi %16, %spills, 24

    br %2
9:%12 = LDRXi %spills, 24 : X0
    ret %12
```

Regalloc Example 3 – Attempt 2

Edge %7 → %2

Critical ϕ :

- ▶ %4 #readers: 0 – broken
- ▶ %5 #readers: 0

```
odd(%0 : X0)
    %spills = alloca 40
    STRXi %0, %spills, 0
    %l3 = LDRXi %spills, 0 : X0; STRXi %l3, %spills, 8
    %c0 = MOVXconst 1 : X0; STRXi %c0, %spills, 16
    %c1 = MOVXconst 0 : X0; STRXi %c1, %spills, 16
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- ↪ Which register to choose?
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- ↪ Are there enough registers? (otherwise: spilling)
- ↪ Which register to choose?
- ↪ Which register to kill and put on the stack?
- ▶ Needs information when value is actually needed

Interlude: Register Allocation Research – Executive Summary

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- ▶ *Tons* of papers exist
- ▶ Papers are academic

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- ▶ Papers often skip over important details
 - ▶ E.g., when spilling – using the value needs another register
 - ▶ E.g., temporary register for shuffling values
- ▶ Additional (ISA) constraints in practice: (incomplete list)
 - ▶ 2-address instructions with destructive source
 - ▶ Fixed registers for specific instructions
 - ▶ Computing the stack address may need yet another register
 - ▶ Different register classes, often just handled independently

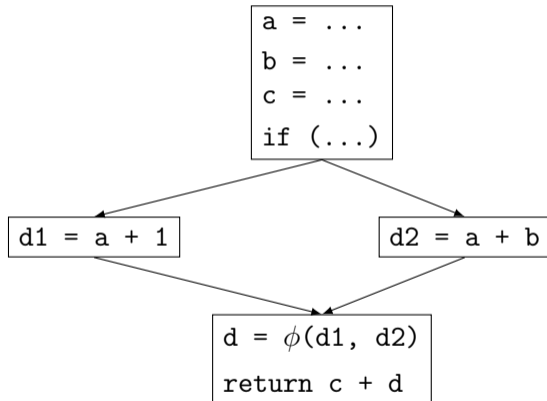
Interlude: Register Allocation Research – Executive Summary

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 - ▶ Different register classes, often just handled independently
- ▶ Implementations even of simple algorithms tend to be large and complex

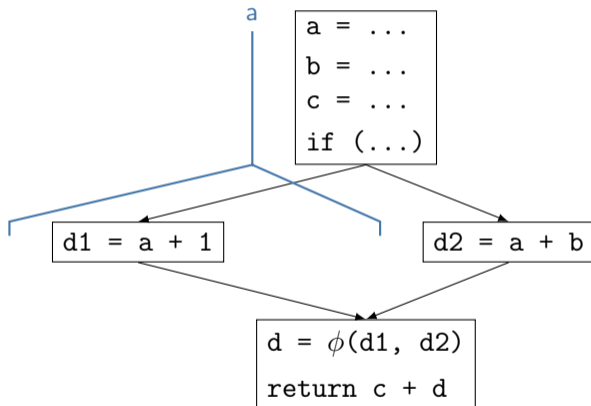
Liveness Analysis – Definitions

- ▶ *Live*: value still used afterwards
 - ▶ After last (possible) use in program flow, the value becomes dead
- ▶ *Live ranges*: set of ranges in program where value is live
 - ▶ Not necessarily contiguous, e.g. in case of branches
- ▶ *Live interval*: over-approximation of live ranges without holes
 - ▶ Depends on block order, reverse post-order often a good choice
- ▶ *Live-in/Live-out*: values live at begin/end of basic block
 - ▶ For ϕ nodes: ϕ is live-in, operands are live-out in predecessors
(Note: different literature uses different definitions)

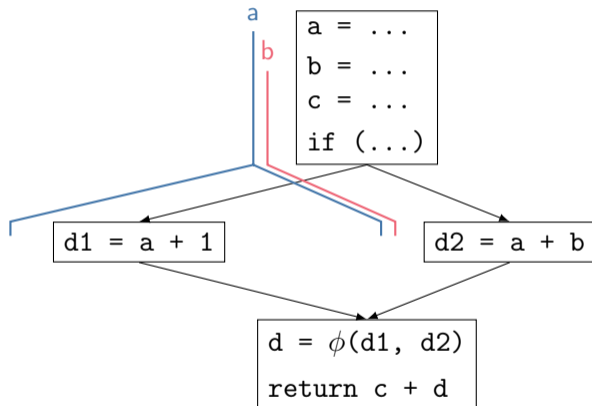
Liveness Analysis – Example



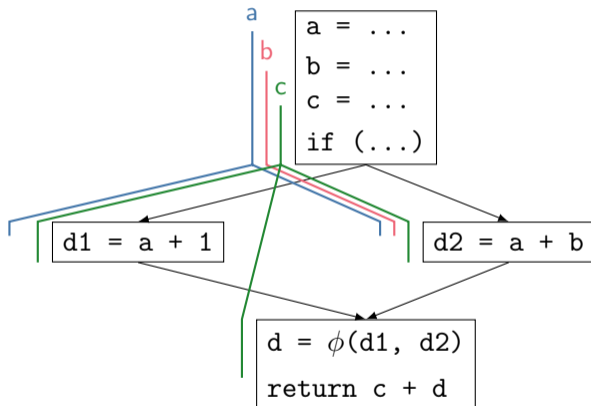
Liveness Analysis – Example



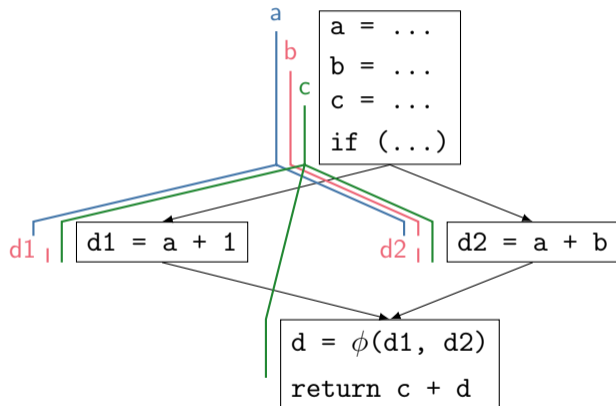
Liveness Analysis – Example



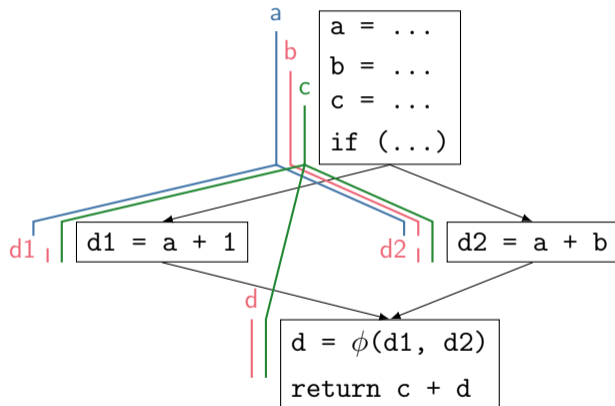
Liveness Analysis – Example



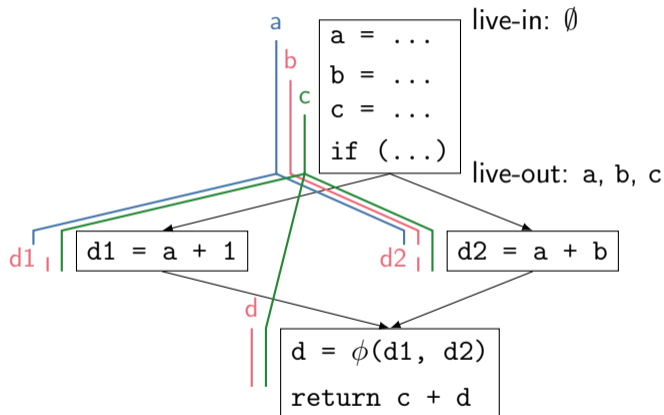
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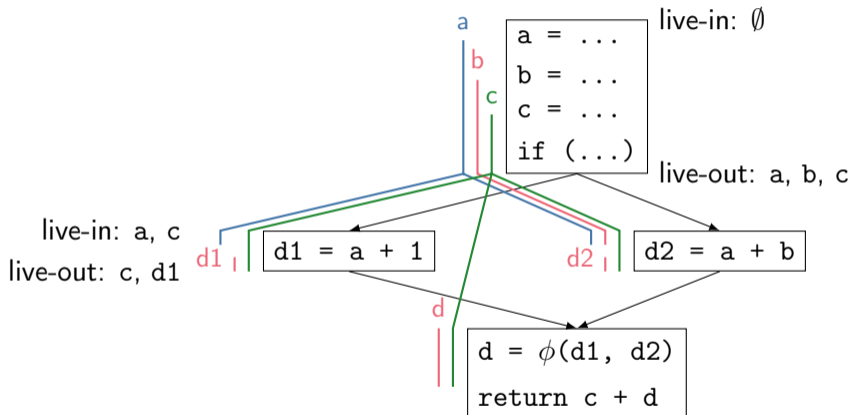
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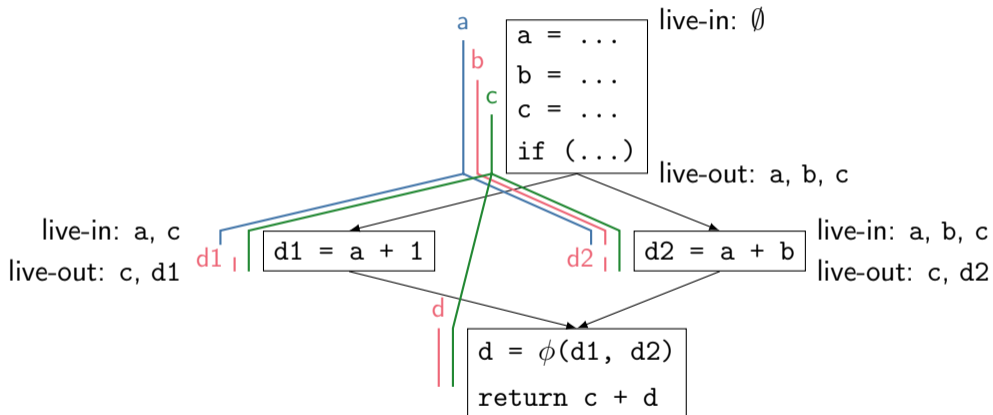
Liveness Analysis – Example



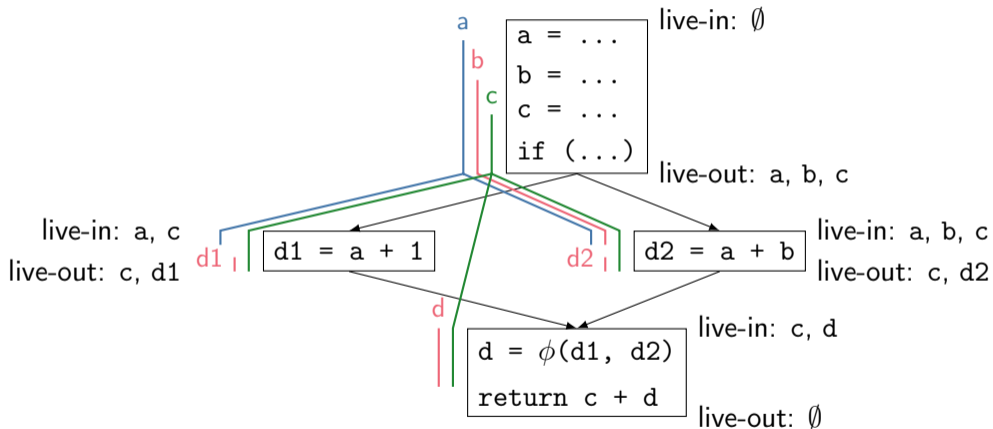
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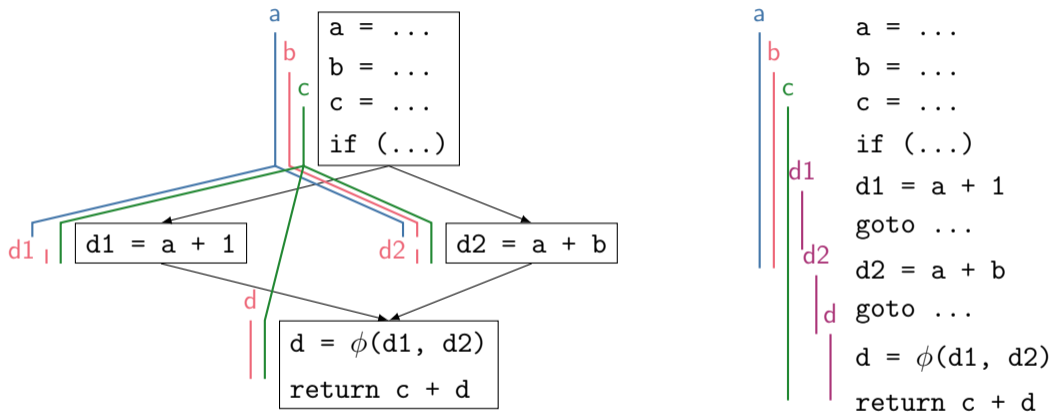
Liveness Analysis – Example



Liveness Analysis – Example



Liveness Analysis – Example – Live Ranges vs. Live Intervals



- Live intervals are substantially worse, but easier to compute

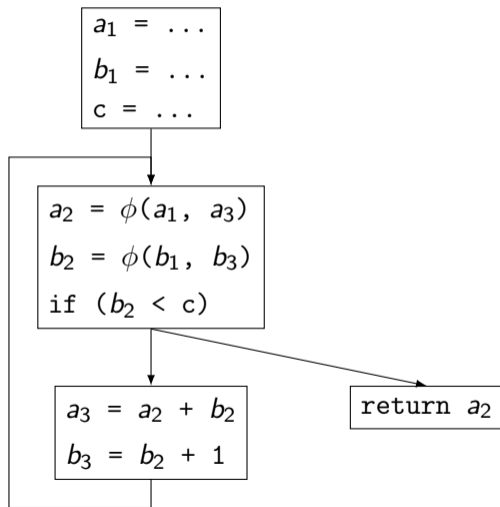
Liveness Analysis – Algorithm³⁶

- ▶ Iterate over blocks in post-order
 - ▶ $live \leftarrow \cup s.liveIn \setminus s.phis, s \in b.successors$
 - ▶ $live \leftarrow live \cup \{\phi.input(b) \mid \phi \in b.successors.phis\}$
 - ▶ $b.liveOut \leftarrow live$
 - ▶ $\forall v \in live : ranges[v].add(b.start, b.end)$
 - ▶ For each non- ϕ instruction $inst$ in reverse order
 - ▶ $live \leftarrow (live \cup inst.ops) \setminus \{inst\}$
 - ▶ $ranges[inst].setStart(inst)$
 - ▶ $\forall op \in inst.ops : ranges[op].add(b.start, inst)$
 - ▶ $b.liveIn \leftarrow live \cup b.phis$
- ▶ Repeat until convergence³⁵

³⁵Reducible graphs: expanding $liveIn$ of loop headers to the entire loop suffices

³⁶Adapted from C Wimmer and M Franz. “Linear scan register allocation on SSA form”. In: CGO. 2010, pp. 170–179.

Liveness Analysis – Example



Register Allocation Decisions (Outline)

- ▶ Question: are there enough registers for all values?
 - ▶ *Register pressure* = number of values live at some point
 - ▶ Register pressure > #registers \Rightarrow move some values to stack (spilling)

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- ▶ Question: when spilling, which values and where to store/reload?
 - ▶ Spilling is expensive, so avoid spilling frequently used values
- ▶ Question: for unspilled values, which register to assign?
 - ▶ Also: respect register constraints, etc.

Register Allocation Strategies

Scan-based

- ▶ Iterate over the program
- ▶ Decide locally what to do
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 - ▶ Edge \Rightarrow live ranges overlap
- ▶ Holistic approach

Register Allocation Strategies

Scan-based

- ▶ Iterate over the program
 - ▶ Decide locally what to do
 - ▶ Greedily assign registers
-
- + Fast, good for straight code
 - Code quality often bad
 - ▶ Used for -O0 and JIT comp.

Graph-based

- ▶ Compute *interference graph*
 - ▶ Nodes are values
 - ▶ Edge \Rightarrow live ranges overlap
 - ▶ Holistic approach
-
- + Often generate good code
 - Expensive, superlinear runtime
 - ▶ Used for optimized code

Linear Scan Register Allocation³⁷

- ▶ Idea: treat whole function as single block
 - ▶ Block order affects quality (but not correctness)
 - ▶ Only consider live intervals without holes
- ▶ Iterate over instructions from top to bottom
- ▶ For operands of instruction in their last use: mark register as free
- ▶ Assign instruction result to new free register
 - ▶ If no free register available: move some value to the stack
 - ▶ Heuristic: value whose liveness ends furthest in future

Linear Scan Register Allocation

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- + low compile-time, simple
- *very* suboptimal code, live intervals grossly over-approximated
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Linear Scan Register Allocation

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- very suboptimal code, live intervals grossly over-approximated
- ▶ What's missing?
 - ▶ Registers to load spilled values
 - ▶ Shuffling of values between blocks
 - ▶ Register constraints (e.g., for instructions or function calls)
- ▶ Other disadvantage: once a value is spilled, it is spilled everywhere
 - ▶ Some other approaches based on lifetime splitting³⁸
- ▶ Function calls: clobber lots of registers

³⁸O Traub, G Holloway, and MD Smith. "Quality and speed in linear-scan register allocation". In: *SIGPLAN 33.5 (1998)*, pp. 142–151. 

Scan-based Register Allocation⁴¹

Iterate over basic blocks³⁹

- ▶ Start with register assignment from predecessor
 - ▶ Multiple predecessors: choose assignment from any one
 - ▶ ϕ -nodes can either reside in registers or on the stack
- ▶ Iterate over instructions top-down
 - ▶ Ensure all instruction operands are in registers
 - ▶ When out of registers: move any value to stack
 - ▶ For operands in their last use: mark register as free
 - ▶ Assign instruction result to new free register
- ▶ Shuffle values back into registers where successor expects them⁴⁰

³⁹Typically: reverse post-order, so most predecessors are seen before successors, except for loops.

⁴⁰Without critical edges, only relevant for blocks with one successor — others are visited afterwards by RPO definition.

⁴¹Mostly following Go: <https://github.com/golang/go/blob/5f7abe/src/cmd/compile/internal/ssa/regalloc.go>

Scan-based Register Allocation – Spilling

What to spill?

Scan-based Register Allocation – Spilling

What to spill?

- ▶ Spill value with furthest use in future⁴²
 - ▶ Frees register for longest time
 - ▶ Requires information on next use to be stored during analysis
 - ▶ But: avoid spilling values computed inside loops (esp. loop-carried dependencies), reloads are fine⁴³
 - ▶ Downside: super-linear runtime

Where to store?

⁴²C Wimmer and H Mössenböck. “Optimized interval splitting in a linear scan register allocator”. In: *VEE*. 2005, pp. 132–141.

⁴³Intel Optimization Reference Manual (Aug. 2023), Assembly/Compiler Coding Rules 38 and 45

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Where to store?

- ▶ Stack, period.
- ▶ Spilling to FP/vector registers. . . occasionally proposed, not used in practice

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Scan-based Register Allocation – Spilling

Where to insert store?

Scan-based Register Allocation – Spilling

Where to insert store?

- ▶ Option 1: spill exactly where required
 - ▶ Downside: multiple spills of same value, many reloads
- ▶ Option 2: spill once, immediately after computation
 - ▶ Later “spills” to the stack are less costly
 - ▶ May lead to spills on code paths that don’t need it
- ▶ Option 3: compute best place using dominator tree
 - ▶ Spill store must dominate all subsequent loads

Scan-based Register Allocation – Register Assignment

- ▶ Merge blocks:

Scan-based Register Allocation – Register Assignment

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 - ▶ High likelihood of reducing the number of stores
 - ▶ Re-loads are pushed into predecessors

Scan-based Register Allocation – Register Assignment

- ▶ Merge blocks: choose predecessor with most values in registers
 - ▶ High likelihood of reducing the number of stores
 - ▶ Re-loads are pushed into predecessors
- ▶ Propagate register constraints bottom-up as hints first
 - ▶ E.g.: call parameters, instruction constraints, assignment for merge block
 - ▶ Reduces number of moves

Graph Coloring Approaches

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- + Considerably better results than greedy algorithms

Graph Coloring Approaches

- + Considerably better results than greedy algorithms
- High run-time, even with heuristics
- ▶ Graph coloring in general is \mathcal{NP} -complete
- ▶ Often used in compilers (e.g., GCC, WebKit)

AD IN2053 “Program Optimization” covers this more formally

Stack Frame Allocation

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- ▶ Optionally setup frame pointer
 - ▶ Required for variably-sized stack frame
Otherwise: cannot access spilled variables or stack parameters
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- ▶ Save callee-saved registers, maybe also link register
- ▶ Optionally add code for stack canary
- ▶ Compute stack frame size and adjust stack pointer
 - ▶ Mainly size of `alloca`s, but needs to respect alignment
 - ▶ Ensure sufficient space for parameters passed on the stack
 - ▶ Ensure stack pointer is sufficiently aligned
- ▶ Stack pointer adjustment *may* be omitted for leaf functions
 - ▶ Some ABIs guarantee a *red zone*

Block Ordering

- ▶ Order blocks to make use of fall-through in machine code
- ▶ Avoid sequences of `b.cond; b`
 - ▶ Sometimes cannot be avoided: conditional branches often have shorter range
- ▶ Block ordering has implications for branch prediction
 - ▶ Forward branches default to not-taken, backward taken
 - ▶ Unlikely blocks placed “out of the way” of the main execution path
 - ▶ Indirect branches are predicted as fall-through

Register Allocation – Summary

- ▶ Map unlimited virtual registers to restricted register set
- ▶ Responsible for:
 - ▶ Assigning registers to values
 - ▶ Deciding which registers to spill to stack
 - ▶ Deciding when to spill/unspill values
- ▶ ϕ -nodes require extra care, esp. for chains and cycles
- ▶ Liveness information is key information for register allocation
- ▶ Scan-based approaches are fast, but lead to suboptimal code
- ▶ Graph coloring yields better results, but is much slower
- ▶ Register allocation/spilling heavily relies on heuristics in practice

Register Allocation – Questions

- ▶ Why is register allocation a difficult problem?
- ▶ How are ϕ -nodes handled during register allocation?
- ▶ What are the two main problems when destructing ϕ -nodes?
- ▶ Why are critical edges problematic and how to deal with them?
- ▶ What are practical constraints for register allocation?
- ▶ How to detect whether a value is still needed at some point?
- ▶ How to compute the live ranges of values in an SSA-based IR?
- ▶ What is the idea of linear scan and what are its practical problems?