Query Optimization
Exercise Session 10

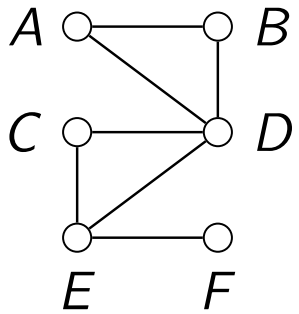
Andrey Gubichev

December 22, 2014

Homework

- Next Homework: Quick Pick
  - Union-Find
  - Union-By-Size
  - Path-Shortening

Quick-Pick this!



Edge	Roots	Union-Find Array	Join Trees
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(A, B)			
(E, F)			
(E, D)			
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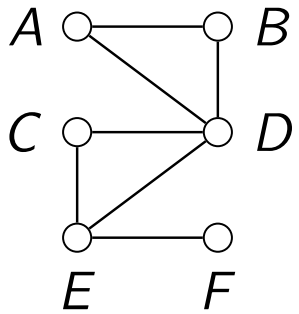

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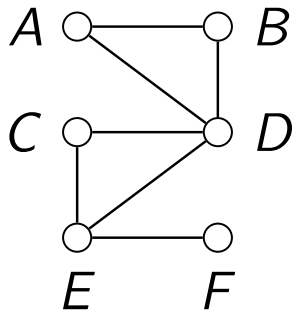

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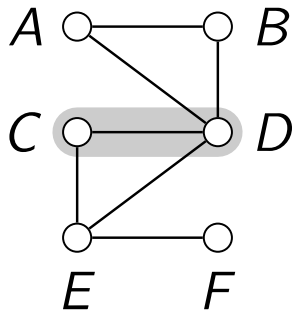

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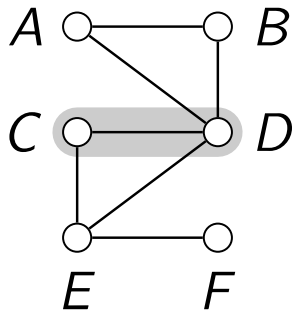

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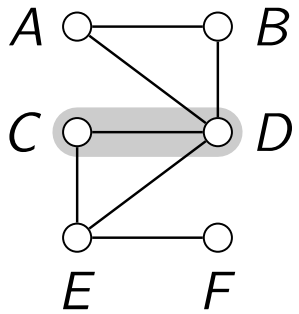

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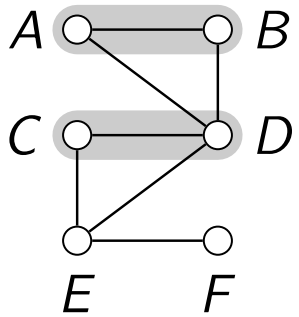

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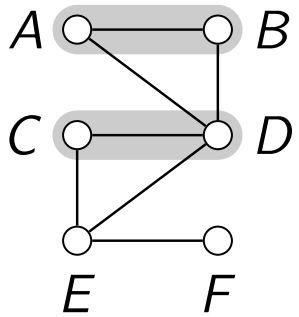

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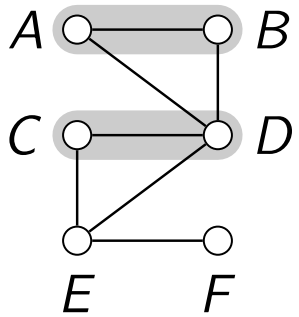

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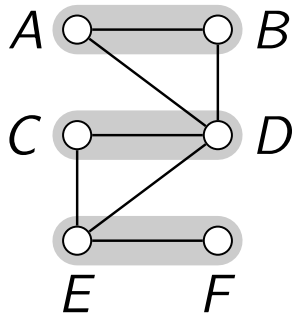

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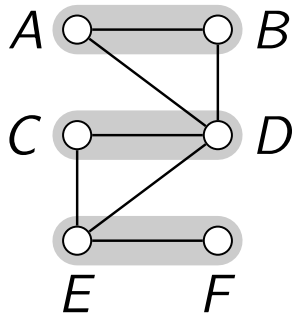

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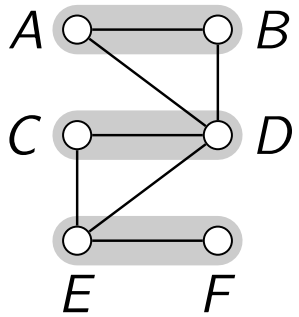

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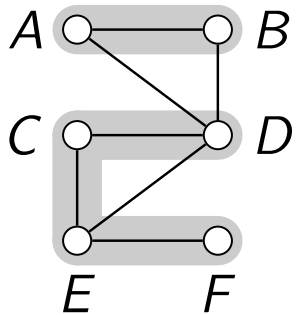

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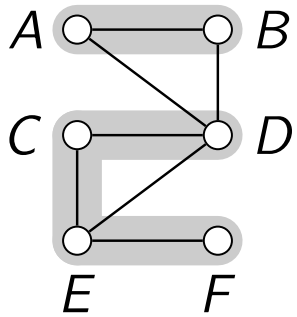

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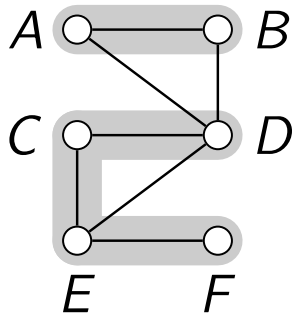

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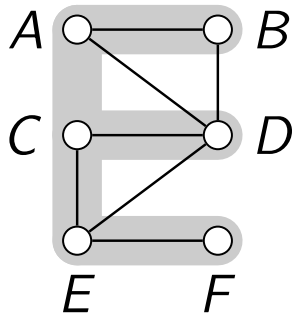

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A: A branch of mathematics concerning the study of finite or countable discrete structures.

Q: What is ?

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Q: What is combinatorics?

Combinatorics 101

Given a set of n elements, how many distinct k-element subsets can be formed?

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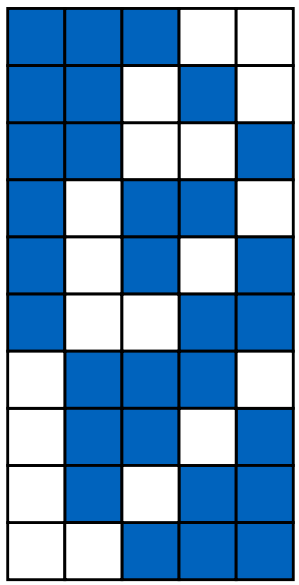
Example: Choose 3 out of 5: $\binom{5}{3} = \frac{5!}{2! \cdot 3!} = \frac{120}{2 \cdot 6} = 10$



Combinatorics 101 revisited

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  $f : (x_1, x_2, \ldots, x_k) \mapsto (x_1 + 0, x_2 + 1, \ldots, x_k + (k - 1))$

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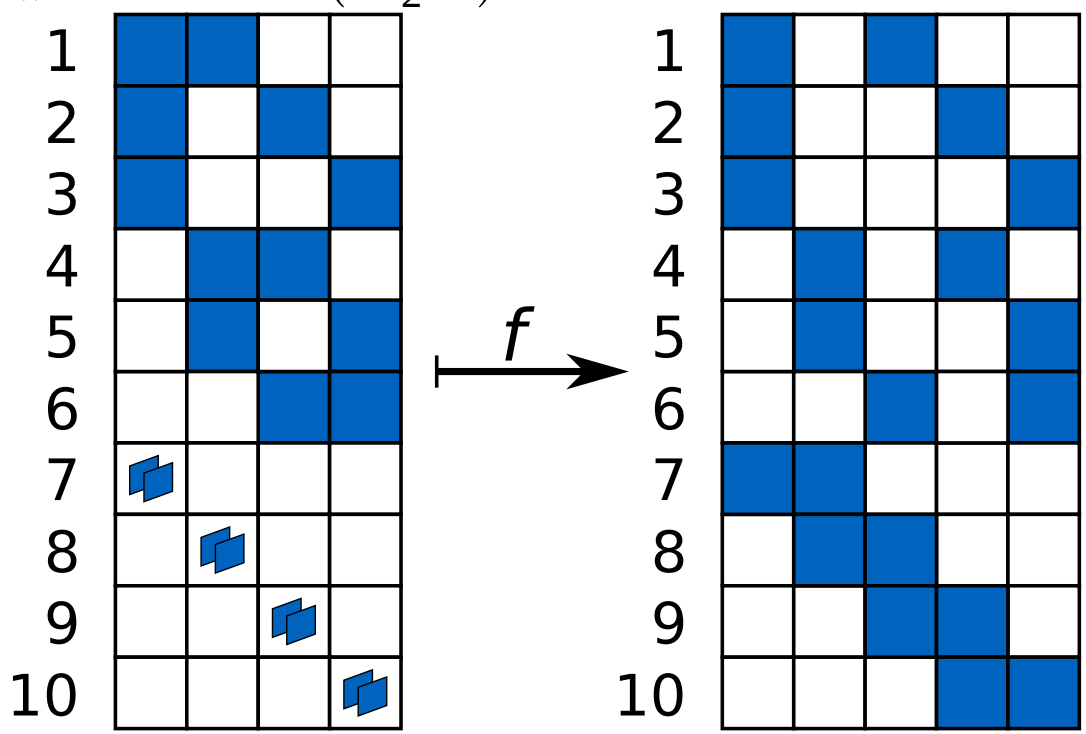
- Example: Choose 2 from 4
  - # sets: $\binom{4}{2}$
  - # multisets: $\binom{4+2-1}{2}$



Info

- Exercises due January 12, 2014.